

CEnT-S Mathematics

Formula Booklet

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CHAPTER 01 — NUMBERS

PAGE 1 — NUMBERS I

Number Sets · Operations · Powers · Roots · Absolute Value

1. Number Sets

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$$

Set	Description
$\mathbb{N}$	Natural numbers
$\mathbb{Z}$	Integers
$\mathbb{Q}$	Rational numbers
$\mathbb{R} \setminus \mathbb{Q}$	Irrational numbers

The inclusion of 0 in $\mathbb{N}$ depends on convention.

2. Fraction Operations

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd} \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \quad \frac{a}{b} = \frac{ak}{bk}$$

All denominators and divisors must be non-zero; $k \neq 0$.

3. Powers

$$a^m a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} \quad (a^m)^n = a^{mn} \quad (ab)^n = a^n b^n \quad a^0 = 1 \quad a^{-n} = \frac{1}{a^n}$$

For integer exponents; all expressions must be defined. Non-zero bases are required where applicable.

4. Roots and Rational Powers

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \sqrt{a^2} = |a|$$

Square-root rules: $a, b \geq 0$, with $b > 0$ for division. For rational powers shown, $a > 0$, $m \in \mathbb{Z}$, $n \in \mathbb{Z}_{>0}$.

5. Absolute Value

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases} \quad |ab| = |a||b|, \quad |-a| = |a|$$

Absolute Value Inequalities

$$|x| \leq a \Leftrightarrow -a \leq x \leq a \quad (a \geq 0)$$

$$|x| \geq a \Leftrightarrow x \leq -a \text{ or } x \geq a \quad (a \geq 0)$$

6. Ordering and Comparison

Operation	Rule
Addition	$a < b \Rightarrow a + c < b + c$
Multiply by $c > 0$	$a < b \Rightarrow ac < bc$
Multiply by $c < 0$	$a < b \Rightarrow ac > bc$

$$0 < a < 1 \Rightarrow a^2 < a < \sqrt{a}$$

CHAPTER 01 — NUMBERS

PAGE 2 — NUMBERS II

Divisibility · Percentages · Proportions · Estimation

1. Division with Remainder

$$N = dq + r, \quad 0 \leq r < d$$

For integers $N \geq 0$, $d > 0$.

2. Prime Factorisation, GCD and LCM

GCD = Greatest Common Divisor · LCM = Least Common Multiple

$$72 = 2^3 \cdot 3^2, \quad 120 = 2^3 \cdot 3 \cdot 5$$

$$\text{GCD}(72, 120) = 24$$

$$\text{LCM}(72, 120) = 360$$

GCD	LCM
Minimum prime exponents	Maximum prime exponents

$$\text{GCD}(a, b) \cdot \text{LCM}(a, b) = ab$$

For positive integers a, b .

3. Divisibility Rules

Divisor	Rule
2	Last digit is even
3	Digit sum divisible by 3
4	Last two digits divisible by 4
5	Last digit is 0 or 5
6	Divisible by both 2 and 3
9	Digit sum divisible by 9
10	Last digit is 0

4. Percentages

$$p\% \text{ of } A = \frac{p}{100} A$$

$$\text{Percentage change} = \frac{B - A}{A} \times 100\%$$

Change	New Value
Increase by $p\%$	$B = A \left(1 + \frac{p}{100} \right)$
Decrease by $p\%$	$B = A \left(1 - \frac{p}{100} \right)$

For percentage change, $A > 0$.

5. Ratios and Proportions

$$a : b = \frac{a}{b}, \quad \frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$$

Direct Proportion

$$y = kx$$

Inverse Proportion

$$xy = k$$

Ratios use nre non-zero denominators; inverse proportion requires $x \neq 0$.

6. Scientific Notation and Estimation

$$x = c \times 10^n, \quad 1 \leq |c| < 10, \quad n \in \mathbb{Z}$$

$$a < b \Rightarrow \sqrt{a} < \sqrt{b} \quad (a, b \geq 0)$$

7. Order of Operations

Brackets $\rightarrow$ Powers $\rightarrow \times, \div \rightarrow +, -$

Operations of equal priority are evaluated from left to right.

CHAPTER 02 — ALGEBRA

PAGE 3 — ALGEBRA I

Algebraic Identities • Factorisation • Rational Expressions

1. Algebraic Identities

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

2. Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

3. Factorisation

Method	Identity
Common factor	$ab + ac = a(b + c)$
Difference of squares	$a^2 - b^2 = (a - b)(a + b)$
Perfect square	$a^2 + 2ab + b^2 = (a + b)^2$
Quadratic factors	$x^2 - (r + s)x + rs = (x - r)(x - s)$

4. Polynomial Roots

$$P(r) = 0 \quad \text{iff} \quad (x - r) \text{ is a factor of } P(x)$$

$$P(x) = a(x - r_1)(x - r_2)$$

The second formula applies to a quadratic polynomial with real roots r_1 , r_2 and $a \neq 0$.

5. Rational Expressions

$$\frac{ab}{ac} = \frac{b}{c} \quad (a \neq 0, c \neq 0)$$

$$\frac{a^2 - b^2}{a - b} = a + b \quad (a \neq b)$$

Excluded values must be determined before simplification.

CHAPTER 02 — ALGEBRA

PAGE 4 — ALGEBRA II

Equations · Quadratic Equations · Solution Sets

1. Linear Equations

$$ax + b = 0 \Rightarrow x = -\frac{b}{a} \quad (a \neq 0)$$

2. Quadratic Equations

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

$$\Delta = b^2 - 4ac$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Discriminant	Real roots
$\Delta > 0$	Two distinct
$\Delta = 0$	One repeated
$\Delta < 0$	No real roots

3. Sum and Product of Roots

$$x_1 + x_2 = -\frac{b}{a}$$

$$x_1 x_2 = \frac{c}{a}$$

4. Vertex Form

$$f(x) = a(x - h)^2 + k$$

$$h = -\frac{b}{2a}, \quad k = -\frac{\Delta}{4a}$$

For $f(x) = ax^2 + bx + c$, $a \neq 0$.

5. Equivalent Equations

$$A = B \text{ iff } A + C = B + C$$

$$A = B \text{ iff } AC = BC \quad (C \neq 0)$$

For variable expressions C , equivalence requires $C \neq 0$ on the permitted domain.

6. Solution Sets and Intervals

Notation	Meaning
(a, b)	$a < x < b$
$[a, b]$	$a \leq x \leq b$
$(-\infty, a]$	$x \leq a$
$[a, \infty)$	$x \geq a$
$A \cap B$	Intersection
$A \cup B$	Union

7. Basic Validity Conditions

Expression	Condition
$\frac{1}{g(x)}$	$g(x) \neq 0$
$\sqrt{g(x)}$	$g(x) \geq 0$
$\log_a g(x)$	$g(x) > 0, a > 0, a \neq 1$

CHAPTER 02 — ALGEBRA

PAGE 5 — ALGEBRA III

Inequalities · Sign Charts · Systems of Equations

1. Linear Inequalities

$$a > 0: \quad ax > b \text{ iff } x > \frac{b}{a}$$

$$a < 0: \quad ax > b \text{ iff } x < \frac{b}{a}$$

2. Quadratic Inequalities

For $f(x) = a(x - r_1)(x - r_2)$, where $a > 0$ and $r_1 < r_2$:

x	$-\infty$	r_1	r_2	$+\infty$	
$f(x)$	+	0	−	0	+
$f(x) > 0$	$\longleftarrow \bigcirc$		$\bigcirc \longrightarrow$		

$$f(x) > 0 \text{ iff } x < r_1 \text{ or } x > r_2$$

$$f(x) < 0 \text{ iff } r_1 < x < r_2$$

For $a < 0$, reverse the signs.

3. Rational Inequalities

$$\frac{P(x)}{Q(x)} > 0 \text{ iff } P(x), Q(x) \text{ have the same sign}$$

$$Q(x) \neq 0$$

Zeros of the denominator are always excluded.

4. Systems of Linear Equations

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

Number of Solutions

Graphical interpretation	Algebraic condition	Solutions
Intersecting lines	$a_1/a_2 \neq b_1/b_2$	One
Parallel distinct lines	$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	None
Coincident lines	$a_1/a_2 = b_1/b_2 = c_1/c_2$	Infinitely many

Ratio conditions assume $a_2, b_2, c_2 \neq 0$. When denominators are zero, compare proportional coefficients directly.

CHAPTER 03 — GEOMETRY

PAGE 6 — GEOMETRY I

Angles · Triangles · Quadrilaterals

1. Angles and Parallel Lines

Straight angle = 180°

Full angle = 360°

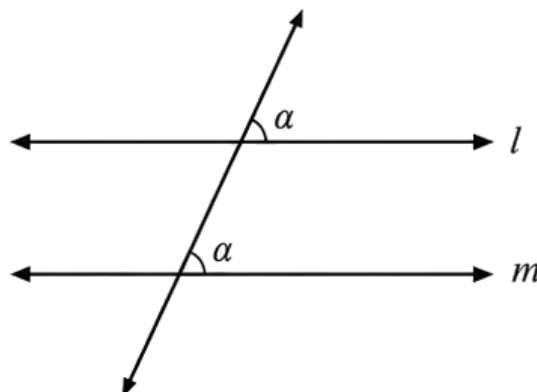
$\alpha + \beta = 90^\circ$ (complementary)

$\alpha + \beta = 180^\circ$ (supplementary)

Corresponding angles are equal

Alternate interior angles are equal

Co-interior angles sum to 180°

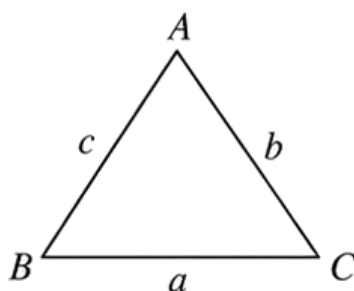


2. Triangles

$A + B + C = 180^\circ$

$|b - c| < a < b + c$

$A_{\text{triangle}} = \frac{1}{2}bh$



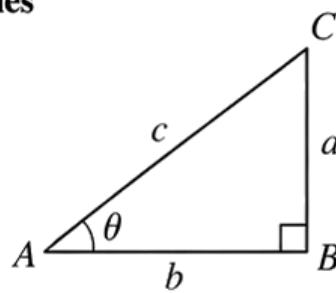
Right Triangles

$a^2 + b^2 = c^2$

$\sin \theta = \frac{a}{c}$

$\cos \theta = \frac{b}{c}$

$\tan \theta = \frac{a}{b}$



c : hypotenuse; a opposite θ ; b adjacent θ

3. Special Right Triangles

Angles	Side ratio
$45^\circ - 45^\circ - 90^\circ$	$1 : 1 : \sqrt{2}$
$30^\circ - 60^\circ - 90^\circ$	$1 : \sqrt{3} : 2$

4. Quadrilateral Properties

Figure	Essential properties
Parallelogram	Opposite sides parallel and equal; diagonals bisect each other
Rectangle	Four right angles; diagonals equal and bisect each other
Rhombus	Four equal sides; diagonals are perpendicular and bisect each other
Square	Four equal sides and four right angles
Trapezoid	At least one pair of parallel sides
Kite	Two pairs of adjacent equal sides; diagonals are perpendicular

Sum of interior angles of a quadrilateral = 360° .

CHAPTER 03 — GEOMETRY

PAGE 7 — GEOMETRY II

Perimeters · Areas · Circles · Similarity

1. Perimeters and Areas

Figure	Perimeter	Area
Square	$4a$	a^2
Rectangle	$2(a + b)$	ab
Triangle	$a + b + c$	$\frac{1}{2}bh$
Parallelogram	$2(a + b)$	bh
Trapezoid	$a + b + c + d$	$\frac{1}{2}(a + b)h$
Rhombus	$4a$	$\frac{1}{2}d_1d_2$
Circle	$2\pi r$	πr^2

For the trapezoid, a and b are the parallel sides. For the rhombus, d_1 and d_2 are the diagonals.

2. Regular Polygons

$$\text{Interior angle} = \frac{(n - 2)180^\circ}{n}$$

$$\text{Exterior angle} = \frac{360^\circ}{n}$$

$$A = \frac{1}{2}Pp$$

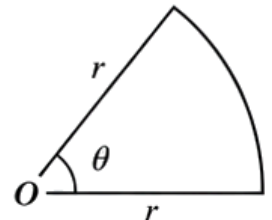
n : number of sides · P : perimeter · p : apothem

3. Circles, Arcs and Sectors

$$s = r\theta$$

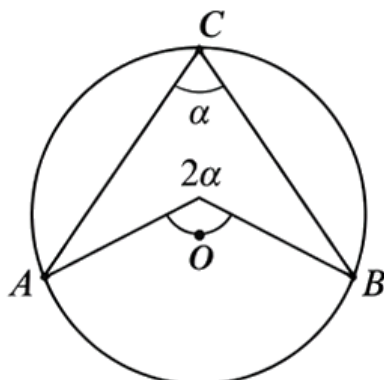
$$A_{\text{sector}} = \frac{1}{2}r^2\theta$$

$$A_{\text{annulus}} = \pi(R^2 - r^2)$$



Angles must be in radians for arc length and sector area formulas.

4. Circle Angle Theorems



$$\angle AOB = 2\angle ACB$$

$$\angle ACB = \frac{1}{2}\angle AOB$$

Angle in a semicircle = 90°

Opposite angles of a cyclic quadrilateral sum to 180° .

5. Similarity

$$\frac{a_2}{a_1} = \frac{b_2}{b_1} = \frac{c_2}{c_1} = k$$

Quantity	Scale factor
Length	k
Area	k^2
Volume	k^3

CHAPTER 03 — GEOMETRY

PAGE 8 — GEOMETRY III

Cartesian Coordinates · Straight Lines · Circles · Regions

1. Distance and Midpoint

For $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

2. Equations of Straight Lines

Slope-intercept form: $y = mx + q$

Point-slope form: $y - y_1 = m(x - x_1)$

Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_2 \neq x_1)$

General form: $ax + by + c = 0$

$$m = -\frac{a}{b} \quad (b \neq 0)$$

3. Parallel and Perpendicular Lines

Relationship	Condition
Parallel	$m_1 = m_2$
Perpendicular	$m_1 m_2 = -1$
Horizontal	$y = k$
Vertical	$x = k$

Slope comparisons require non-vertical lines. A horizontal line is perpendicular to a vertical line.

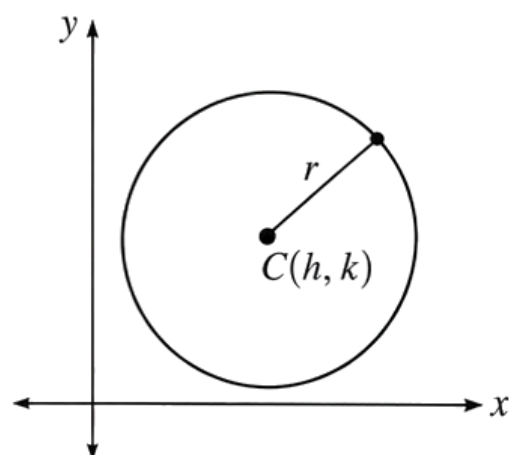
4. Intersection of Two Lines

For $y = m_1x + q_1$ and $y = m_2x + q_2$:

$$x = \frac{q_2 - q_1}{m_1 - m_2} \quad (m_1 \neq m_2)$$

$$y = m_1x + q_1$$

5. Equation of a Circle



$$(x - h)^2 + (y - k)^2 = r^2$$

Centre: (h, k)

Radius: $r > 0$

General form:

$$x^2 + y^2 + Dx + Ey + F = 0$$

$$C = \left(-\frac{D}{2}, -\frac{E}{2} \right)$$

$$r = \sqrt{\frac{D^2 + E^2}{4} - F}$$

For a non-degenerate circle:

$$\frac{D^2 + E^2}{4} - F > 0$$

6. Regions Defined by Inequalities

Inequality	Region
$(x - h)^2 + (y - k)^2 < r^2$	Inside the circle
$(x - h)^2 + (y - k)^2 > r^2$	Outside the circle
$y > mx + q$	Above the line
$y < mx + q$	Below the line

Use $\leq$ or $\geq$ to include the boundary. Systems represent the intersection of their solution regions.

CHAPTER 03 — GEOMETRY

PAGE 9 — GEOMETRY IV

Solid Geometry · Surface Areas · Volumes

1. Prisms and Cylinders

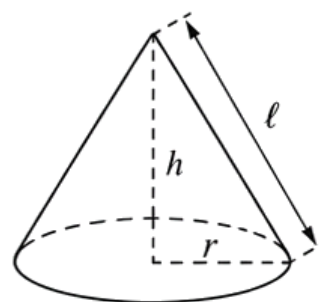
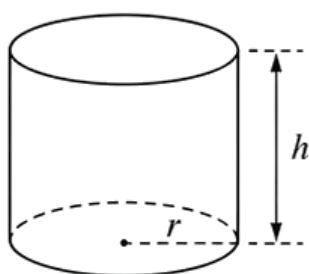
Solid	Volume V	Total surface area A_T
Cube	s^3	$6s^2$
Rectangular prism	abc	$2(ab + ac + bc)$
Right prism	Bh	$2B + Ph$
Right circular cylinder	$\pi r^2 h$	$2\pi r^2 + 2\pi rh$

2. Pyramids, Cones and Spheres

Solid	Volume V	Total surface area A_T
Pyramid	$\frac{1}{3} Bh$	$B + A_L$
Regular pyramid	$\frac{1}{3} Bh$	$B + \frac{1}{2} P\ell$
Right circular cone	$\frac{1}{3} \pi r^2 h$	$\pi r^2 + \pi r\ell$
Sphere	$\frac{4}{3} \pi r^3$	$4\pi r^2$

B : base area · P : base perimeter · h : perpendicular height · ℓ : slant height · A_L : lateral area

3. Essential Solid Diagrams



4. Height, Slant Height and Diagonal

$$\ell_{\text{cone}} = \sqrt{r^2 + h^2}$$

$$d_{\text{rectangular prism}} = \sqrt{a^2 + b^2 + c^2}$$

5. Additional Properties

Solid	Essential property
Prism	Two parallel, congruent polygonal bases
Pyramid	Polygonal base; triangular lateral faces meet at one apex
Cylinder	Two parallel, congruent circular bases
Cone	Circular base; one apex
Sphere	All surface points are equidistant from the centre

For a regular pyramid, ℓ is the altitude of a lateral face, not the perpendicular height h .

CHAPTER 04 — FUNCTIONS

PAGE 10 — FUNCTIONS I

Domain · Range · Properties · Composition · Inverse

1. Functions and Their Properties

$$f : A \rightarrow B, \quad y = f(x)$$

Concept	Mathematical condition
Domain	$x \in D_f$
Range	$\{f(x) : x \in D_f\}$
Zero / Root	$f(x) = 0$
Increasing	$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$
Decreasing	$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$
Even function	$f(-x) = f(x)$
Odd function	$f(-x) = -f(x)$

Strictly increasing or decreasing functions use $<$ or $>$.

2. Domain Restrictions

Expression	Required condition
$\frac{1}{g(x)}$	$g(x) \neq 0$
$\sqrt{g(x)}$	$g(x) \geq 0$
$\log_a g(x)$	$g(x) > 0, \quad a > 0, \quad a \neq 1$

All restrictions must hold simultaneously.

3. Symmetry

Even Function

$$f(-x) = f(x)$$

Symmetry about the y-axis.

Odd Function

$$f(-x) = -f(x)$$

Symmetry about the origin.

4. Composition of Functions

$$f(g(x))$$

$$f(g(x)) \neq g(f(x)) \quad \text{in general}$$

Both $g(x)$ and $f(g(x))$ must be defined.

5. Inverse Functions

$$f^{-1}(f(x)) = x$$

$$f(f^{-1}(x)) = x$$

$$D_{f^{-1}} = R_f, \quad R_{f^{-1}} = D_f$$

An inverse exists on the chosen domain when f is one-to-one.

Graph of an Inverse Function

$$(a, b) \longleftrightarrow (b, a)$$

Graph of f^{-1} is the reflection of the graph of f in $y = x$.

CHAPTER 04 — FUNCTIONS

PAGE 11 — FUNCTIONS II

Elementary Functions and Their Graphs

1. Linear and Quadratic Functions

Linear Function

$$f(x) = mx + q$$

$$\text{slope} = m$$

$$x\text{-intercept} = -\frac{q}{m} \quad (\text{for } m \neq 0)$$

Quadratic Function

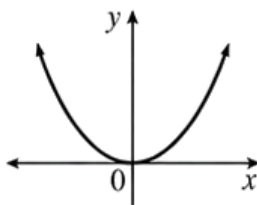
$$f(x) = ax^2 + bx + c \quad (a \neq 0)$$

$$x_v = -\frac{b}{2a}$$

$$y_v = -\frac{\Delta}{4a} \quad (\Delta = b^2 - 4ac)$$

2. Essential Parent Functions

Quadratic Function

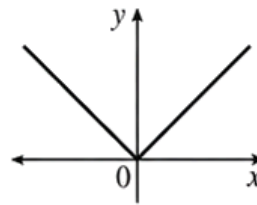


$$f(x) = x^2$$

$$D = \mathbb{R}$$

$$R = [0, \infty)$$

Absolute Value Function

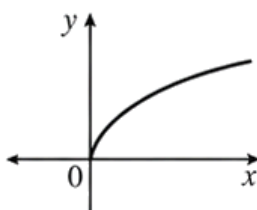


$$f(x) = |x|$$

$$D = \mathbb{R}$$

$$R = [0, \infty)$$

Square Root Function

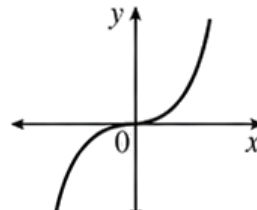


$$f(x) = \sqrt{x}$$

$$D = [0, \infty)$$

$$R = [0, \infty)$$

Cubic Function

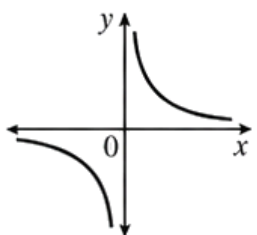


$$f(x) = x^3$$

$$D = \mathbb{R}$$

$$R = \mathbb{R}$$

Reciprocal Function



$$f(x) = \frac{1}{x}$$

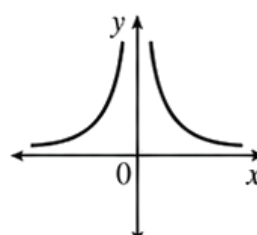
$$D = \mathbb{R} \setminus \{0\}$$

$$R = \mathbb{R} \setminus \{0\}$$

$$\text{Vertical asymptote: } x = 0$$

$$\text{Horizontal asymptote: } y = 0$$

Reciprocal Square Function



$$f(x) = \frac{1}{x^2}$$

$$D = \mathbb{R} \setminus \{0\}$$

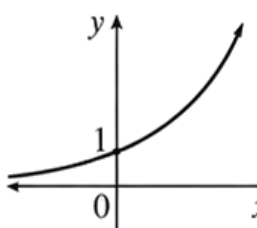
$$R = (0, \infty)$$

$$\text{Vertical asymptote: } x = 0$$

$$\text{Horizontal asymptote: } y = 0$$

3. Exponential and Logarithmic Functions

Exponential Function



$$f(x) = a^x$$

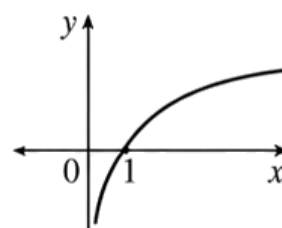
$$D = \mathbb{R}$$

$$R = (0, \infty)$$

$$\text{Passes through } (0, 1).$$

$$\text{Horizontal asymptote: } y = 0.$$

Logarithmic Function



$$f(x) = \log_a x$$

$$D = (0, \infty)$$

$$R = \mathbb{R}$$

$$\text{Passes through } (1, 0).$$

$$\text{Vertical asymptote: } x = 0.$$

$$a > 0, \quad a \neq 1.$$

Increasing for $a > 1$, decreasing for $0 < a < 1$.

CHAPTER 04 — FUNCTIONS

PAGE 12 — FUNCTIONS III

Graph Transformations · Point Mapping · Graphical Solutions

1. Graph Transformations

Transformation	Meaning
$f(x) + k$	Graph shifted up by k .
$f(x) - k$	Graph shifted down by k .
$f(x - h)$	Graph shifted right by h .
$f(x + h)$	Graph shifted left by h .
$-f(x)$	Reflection in the x -axis.
$f(-x)$	Reflection in the y -axis.
$cf(x)$	Vertical scaling by $ c $.
$f(cx)$	Horizontal scaling by $1/ c $.

For $c \neq 0$, negative c also causes reflection in the corresponding axis.

2. Point Mapping

If (a, b) lies on $y = f(x)$, then

New function	Mapped point
$f(x - h)$	$(a + h, b)$
$f(x) + k$	$(a, b + k)$
$cf(x)$	(a, cb)
$f(cx)$	$(\frac{a}{c}, b)$

This table is essential and must remain prominent.

3. Combined Transformations

$$g(x) = Af(B(x - h)) + k$$

Point mapping: $(a, b) \leftrightarrow \left(h + \frac{a}{B}, Ab + k\right)$, where $A, B \neq 0$.

4. Graphical Equations

$f(x) = 0$ Solutions are the x -intercepts of the graph.

$f(x) = g(x)$ Solutions are the x -coordinates of the intersection points.

5. Graphical Inequalities

Inequality	Graphical meaning
$f(x) > 0$	Graph is above the x -axis.
$f(x) < 0$	Graph is below the x -axis.
$f(x) > g(x)$	Graph of f is above the graph of g .
$f(x) \geq g(x)$	Graph of f is above or on the graph of g .

6. Intercepts and Asymptotes

Concept	Equation / Description
x -intercepts	$f(x) = 0$
y -intercept	$(0, f(0))$, if defined.
vertical asymptote	$x = a$
horizontal asymptote	$y = b$

CHAPTER 05 — EXPONENTIALS & LOGARITHMS

PAGE 13 — EXPONENTIALS & LOGARITHMS I

Definitions · Essential Values · Logarithm Laws

1. Exponential and Logarithmic Forms

$$\log_a x = y \quad \text{iff} \quad a^y = x$$

$$a > 0, \quad a \neq 1, \quad x > 0$$

2. Essential Values

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log_a (a^x) = x$$

$$a^{\log_a x} = x$$

3. Logarithm Laws

Operation	Property
Product	$\log_a (xy) = \log_a x + \log_a y$
Quotient	$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$
Power	$\log_a (x^r) = r \log_a x$
Change of base	$\log_a x = \frac{\log_b x}{\log_b a}$

For $x, y > 0$, real r , and valid bases $a, b > 0, a, b \neq 1$.

4. Natural and Common Logarithms

$$\ln x = \log_e x$$

$$\log x = \log_{10} x$$

In this booklet, $\log x$ denotes the common logarithm.

5. Logarithmic Domain Conditions

Expression	Required condition
$\log_a f(x)$	$f(x) > 0$
$\log_{g(x)} f(x)$	$f(x) > 0, \quad g(x) > 0, \quad g(x) \neq 1$
$\frac{1}{\log_a f(x)}$	$f(x) > 0, \quad f(x) \neq 1$

For fixed-base expressions, $a > 0, a \neq 1$.

6. Important Distinction

$$\log_a (x + y) \neq \log_a x + \log_a y$$

In general, when the expressions are defined.

CHAPTER 05 — EXPONENTIALS & LOGARITHMS

PAGE 14 — EXPONENTIALS & LOGARITHMS II

Equations · Inequalities · Comparisons

1. Exponential Equations

$$\begin{aligned} a^{f(x)} &= a^{g(x)} & \text{iff} & & f(x) &= g(x) \\ a^{f(x)} &= b & \text{iff} & & f(x) &= \log_a b \end{aligned}$$

$$a > 0, \quad a \neq 1, \quad b > 0.$$

2. Logarithmic Equations

$$\begin{aligned} \log_a f(x) &= \log_a g(x) & \text{iff} & & f(x) &= g(x) \\ \log_a f(x) &= c & \text{iff} & & f(x) &= a^c \end{aligned}$$

Logarithm arguments must be positive; $a > 0, \quad a \neq 1$.

3. Exponential Inequalities

Base	Comparison
$a > 1$	$a^u < a^v \quad \text{iff} \quad u < v$
$0 < a < 1$	$a^u < a^v \quad \text{iff} \quad u > v$

4. Logarithmic Inequalities

Base	Comparison
$a > 1$	$\log_a u < \log_a v \quad \text{iff} \quad u < v$
$0 < a < 1$	$\log_a u < \log_a v \quad \text{iff} \quad u > v$

For logarithmic inequalities, $u, v > 0$.

5. Comparison and Estimation

For $a > 1$:

$$a^m < x < a^n \quad \text{iff} \quad m < \log_a x < n$$

For $0 < a < 1$:

$$a^m > x > a^n \quad \text{iff} \quad m < \log_a x < n$$

Here $m < n$ and $x > 0$.

CHAPTER 06 — COMBINATORICS & PROBABILITY

PAGE 15 — COMBINATORICS & PROBABILITY

Counting Principles · Permutations · Combinations · Probability

1. Fundamental Counting Principles

- A. Multiplication Rule

$N = n_1 n_2 \cdots n_k$

Successive choices.
- B. Addition Rule

$N = n_1 + n_2 + \cdots + n_k$

Mutually exclusive alternatives.

2. Counting Methods

Selection type	Order matters?	Repetition?	Formula
Permutations	Yes	No	$n!$
Arrangements	Yes	No	$\frac{n!}{(n - k)!}$
Arrangements with repetition	Yes	Yes	n^k
Combinations	No	No	$\binom{n}{k} = \frac{n!}{k!(n - k)!}$

$0! = 1$

3. Basic Probability

$$P(A) = \frac{|A|}{|\Omega|}$$

$0 \leq P(A) \leq 1, \quad P(\Omega) = 1, \quad P(\emptyset) = 0$

Assumes equally likely outcomes in a finite sample space.

4. Complementary Events

$$P(A^c) = 1 - P(A)$$

5. Union and Intersection of Events

- A. Disjoint Events

$P(A \cup B) = P(A) + P(B)$
- B. Independent Events

$P(A \cap B) = P(A)P(B)$

6. Essential Distinction

Disjoint events	Independent events
Cannot occur together	Occurrence of one does not change the probability of the other
$A \cap B = \emptyset$	Neither event affects probability of the other

Disjoint events with positive probabilities cannot be independent.

CHAPTER 07 — BASIC STATISTICS

PAGE 16 — BASIC STATISTICS

Data · Measurement Scales · Frequencies · Averages · Charts

1. Data and Variables

Type	Description
Qualitative	Categories or labels
Quantitative — Discrete	Countable numerical values
Quantitative — Continuous	Measurements within an interval

2. Measurement Scales

Scale	Key feature	Example
Nominal	Categories without order	Blood type
Ordinal	Ordered categories	Satisfaction rating
Interval	Equal intervals, no true zero	Temperature (°C)
Ratio	Equal intervals, true zero	Height, mass

3. Frequencies

Measure	Formula
Absolute frequency	f_i
Relative frequency	$r_i = \frac{f_i}{N}$
Percentage frequency	$r_i \times 100\%$

$$N = \sum_i f_i, \quad \sum_i r_i = 1$$

N : total observations · f_i : frequency of category or value i .

4. Measures of Central Tendency

Arithmetic Mean

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Weighted Mean

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

Combined Mean

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Median

For ordered data $x_{(1)} \leq \cdots \leq x_{(n)}$:

$$\text{Median} = \begin{cases} x_{((n+1)/2)}, & n \text{ odd} \\ \frac{x_{(n/2)} + x_{(n/2+1)}}{2}, & n \text{ even} \end{cases}$$

For frequency tables, use cumulative counts to locate the middle observation(s).

Mode

Mode = most frequent value

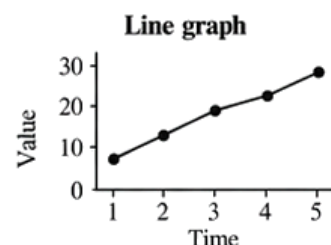
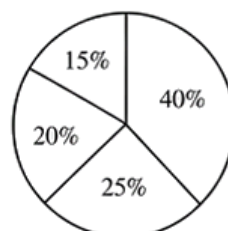
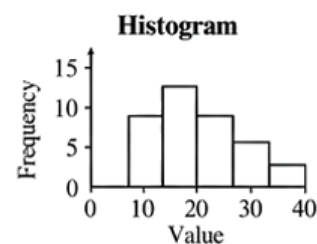
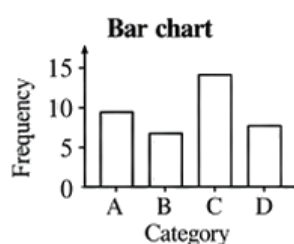
A dataset may have more than one mode.

5. Data Displays

Graph	Essential characteristic
Bar chart	Separated bars; categories or discrete values
Histogram	Adjacent bins; numerical intervals
Pie chart	Sectors represent proportions of a whole
Line graph	Shows changes or trends, often over time

Pie Chart

$$\theta = \frac{f_i}{N} \times 360^\circ$$



For histograms with unequal bin widths, bar area—not height alone—represents frequency.